Chapter 3

Time Value of Money

Future Value

- Compound Interest
  - Earning Interest on Interest
- Basic Components
  - PV = Initial Deposit
  - i = Interest Rate
  - n = Number of Years
  - FV_n = Value at a specified future period

Future Value

General equation:

$$FV_n = PV(1+i)^n$$

Future Value

- Example 3-1:
  - What is the value at the end of year 5 of $100 deposited today if the interest rate is 10% compounded annually?

$$FV_5 = \$100(1.10)^5$$
$$= \$100(1.61051)$$
$$= \$161.05$$

Future Value

- Example 3-1 Using a Financial Calculator:

PV = $100

n = 5

i = 10

CPT **FV** = $161.05

Future Value

- Semi-Annual Compounding
  - In Example 3-1, what if interest were paid semi-annually instead of annually?
    - There would be two compounding periods in each year.
    - There would be a periodic rate to match the multiple compounding periods.
    - The time period would be doubled.
    - *Most importantly, the future value would be higher. Additional compounding periods will effect the final result.*

Future Value

Our general equation becomes:

$$FV_n = PV\left[1+\frac{i}{m}\right]^{n\cdot m}$$

where m = number of compounding intervals in a year

Future Value

- $\frac{i}{m}$ is also called the period rate
- For Example 1:

$$FV_5 = 100\left[1 + \frac{.10}{2}\right]^{5 \cdot 2}$$

$= 100(1.62889)$

$= \$162.89$

Future Value

- Two alternatives for multiple compounding periods and most financial calculators
  - You can change P/Y to the number of compounding periods
    - Example: Change P/Y to 2 for semiannual compounding
  - You can enter a periodic rate
    - Example: Enter i/2 as the interest rate for semiannual compounding

Future Value

- If you change P/Y to 2, then

PV = $100

n = 10

i = 10

PMT = $0

CPT **FV** = $162.89

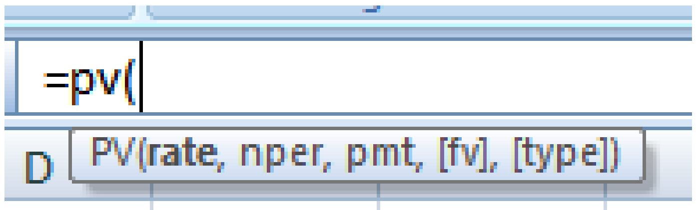
Future Value

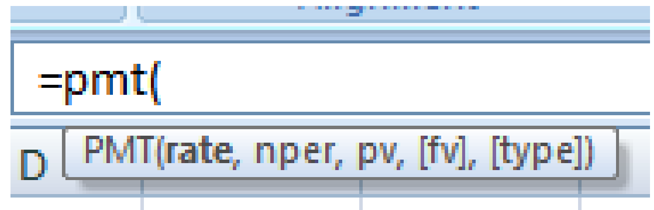
- Notice the difference in Future Value when multiple compounding periods are used:

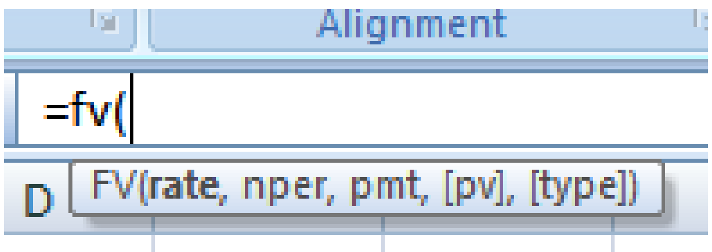
$162.89 vs. $161.05

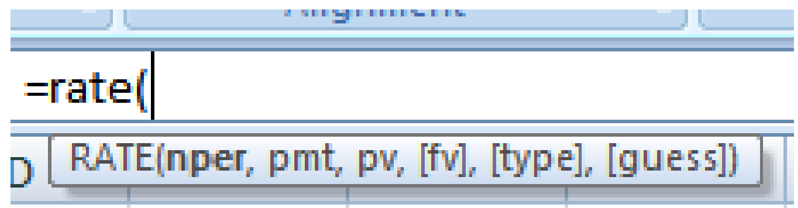
- This shows the effect of earning interest on interest. The more compounding periods there are per year, the higher the future value will be.

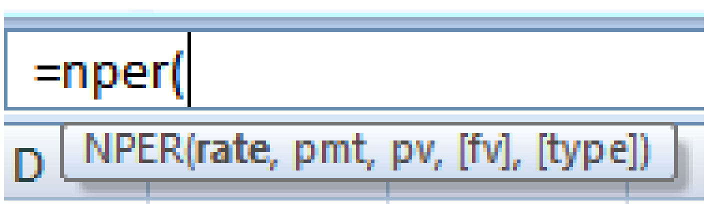
Spreadsheet Functions











For complex analysis, Excel is much better than the financial calculator. It is far more powerful and capable.

Present Value

- Discounting: Converting Future Cash Flows to the Present
- General Equation

$$PV = FV_n \frac{1}{(1+i)^n}$$

Present Value

- Example 3-2:
  - What is the value today of $2,000 you will receive in year 3 if the interest rate is 8% compounded annually?

$$PV = 2000\left[\frac{1}{(1.08)^3}\right]$$

= 2000(.79383)

= $1587.66

Present Value

- Example 3-2 Using a Financial Calculator:

FV = $2000

n = 3

i = 8

CPT **PV** = $1587.66

Present Value

- Example 3-2 with 8% Compounded Monthly
- Mathematically:

$$PV = FV_n \left[\frac{1}{(1 + \frac{i}{m})^{n \cdot m}} \right]$$

$$PV = FV_n \left[MPVIF_{i,n \cdot m} \right]$$

Present Value

$$PV = 2000\left[\frac{1}{\left(1+\frac{.08}{12}\right)^{12\cdot 3}}\right]$$

$= 2000(.78725)$

$= \$1574.51$

Present Value

- If P/Y is changed to 12

FV = $2000

n = 36

i = 8

PMT = $0

CPT **PV** = $1574.51

Annuity

- An annuity is a level cash flow stream.
- One needs to establish not only the length but also if the payments are at the beginning or the end of the month.
- Ordinary Annuity
  - Cash flows begin one period from today
- Annuity Due
  - Cash flows begin immediately

Annuity: Future Value

- General Equation:

$$FV = P(1+i)^{n\text{-}1} + P(i+i)^{n\text{-}2} + \ldots + P$$

$$FV = P \cdot \sum_{t=1}^{n-1} (1+i)^t + P$$

Annuity: Future Value

- Example 3-3:
  - What is the future value of a 5-year ordinary annuity with annual payments of $200, evaluated at a 15% interest rate?

$$FVA = 200 \cdot \frac{(1+.15)^5 - 1}{.15}$$

$$= 200(6.74238)$$

$$= \$1348.48$$

Annuity: Future Value

- Using the Financial Calculator:

PMT	= $200
n	= 5
i	= 15
PV	= $0
CPT **FV**	= $1,348.48

Annuity: Future Value

- For Example 3-3, if payments were to be received monthly
- Mathematically:

$$FV = P \cdot \left[1 + \frac{i}{12}\right]^{n \cdot m - 1} + P \cdot \left[1 + \frac{i}{12}\right]^{n \cdot m - 2} + \ldots + P$$

$$FV = P \cdot \sum_{t=1}^{n \cdot m - 1} \left[1 + \frac{i}{12}\right]^{t} + P$$

Annuity: Future Value

$$FV = 200 \cdot \left[\frac{\left(1 + \frac{.15}{12}\right)^{5 \cdot 12} - 1}{\frac{.15}{12}} \right]$$

$= 200(88.5745)$

$= \$17,714.90$

Annuity: Future Value

- Using the Financial Calculator, if P/Y = 12

PMT		= $200
n		= 60
i		= 15
PV		= $0
CPT	**FV**	= $17,714.90

Annuity: Present Value

- General Equation:

$$PV = PMT \cdot \frac{1}{(1+i)^1} + PMT \cdot \frac{1}{(1+i)^2} + \ldots + PMT \cdot \frac{1}{(1+i)^n}$$

$$PV = PMT \cdot \sum_{t=1}^{n} \frac{1}{(1+i)^t}$$

Annuity: Present Value

- Example 3-4:
  - If you had the opportunity to purchase a $500 per year, ten-year annuity, what is the most you would pay for it? The interest rate is 8%.

$$PVA = 500 \cdot \frac{(1 - \frac{1}{1.08^{10}})}{.08}$$

= 500(6.7100)

= $3355.00

Annuity: Present Value

- Using the Financial Calculator:

PMT = $500

n = 10

i = 8

FV = $0

CPT **PV** = $3,355.00

Annuity: Present Value

- For Example 3-4, if Payments were to be Received Monthly
- Mathematically:

$$PV = P\left[\frac{1}{1+\frac{i}{12}}\right]^{1} + P\left[\frac{1}{1+\frac{i}{12}}\right]^{2} + \ldots + P\left[\frac{1}{1+\frac{i}{12}}\right]^{12\cdot n}$$

Annuity: Present Value

$$PVA = \$500 \cdot \left[\frac{1 - \dfrac{1}{(1 + \frac{.08}{12})^{120}}}{\dfrac{.08}{12}} \right]$$

= \$500(82.4215)

= \$41,210.74

Annuity: Present Value

- Using the Financial Calculator, if P/Y = 12

PMT = $500

n = 120

i = 8

FV = $0

CPT **PV** = $41,210.74

Time Value of Money – Extensions

- Given the basic equations that we have discussed, we can solve for any missing single variable.
- Some common applications
  - Solve for the interest rate
  - Compute payments to accumulate a future sum
  - Compute payments to amortize a loan.

Time Value of Money – Extensions

- Rate of Return or Discount Rate
- Example 3-5:
  - Reed & Portland Trucking is financing a new truck with a loan of $10,000, to be repaid in 5 annual end-of-year installments of $2,504.56. What annual interest rate is the company paying?

Time Value of Money – Extensions

- Set P/Y = 1:

PV = $10,000

n = 5

PMT = ($2504.56)

FV = $0

CPT **i** = 8%

Time Value of Money – Extensions

- Example 3-6:
  - A bank makes a $100,000 loan and will receive payments of $805 each month for 30 years as repayment. What is the rate of return to the bank for making this loan?
    - This is also the cost to the borrower.

Time Value of Money – Extensions

- Set P/Y = 12

PMT		= $805
n		= 360
PV		= ($100,000)
FV		= $0
CPT	**i**	= 9%

Time Value of Money – Extensions

- Example 3-7: Accumulating a Future Sum
  - An individual would like to purchase a home in five (5) years. The individual will accumulate enough money for a $20,000 down payment by making equal monthly payments to an account that is expected to earn 12% annual interest compounded monthly. How much are the equal monthly payments?

Time Value of Money – Extensions

- Set P/Y = 12

FV	= $20,000
n	= 60
PV	= $0
i	= 12
CPT **PMT**	= $244.89

Time Value of Money – Extensions

- The Power of Compounding
- In Example 3-7, our saver deposited

  $244.89 x 60 = $14,693.40
- Interest Earned was

  $20,000 - $14,693.40 = $5,306.60

Time Value of Money – Extensions

- Example 3-8: Amortizing a Loan
  - Your company would like to borrow $100,000 to purchase a piece of machinery. Assume that you can make one payment at the end of each year, the term is 15 years, and interest rate is 7%. What is the amount of the annual payment?

Time Value of Money – Extensions

- Set P/Y = 1:

PV = $100,000

n = 15

FV = $0

i = 7

CPT **PMT** = $10979.46

Equivalent Nominal Annual Rate

- ENAR = Equivalent Nominal Annual Rate
- EAY = Effective Annual Yield

$$ENAR = \left[(1 + EAY)^{\frac{1}{m}} - 1 \right] \cdot m$$

Your Lottery Prize

What exactly is my prize?

If you won a Lotto or Mega Millions jackpot, the prize you have won (or will share) is a multi-million dollar jackpot prize. Your jackpot prize will be paid according to the choice you made at the time you purchased your ticket. Lotto and Mega Millions jackpot prizes can be paid in one lump sum payment equal to the current cash value of the advertised jackpot or in 26 annual payments over 25 years. Lotto jackpot payments are graduated payments, increasing in value each year. Mega Millions jackpot payments are paid in equal payments annually.

Instant Game jackpot prizes may be paid in one lump sum payment equal to the established cash value of the jackpot prize or annually usually over a 20-year period or quarterly in February, May, August and November, depending on the prize structure of the game.